Locating Hubs:

an overview of models and potential applications

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What is hub location?

How to model hub location problems (HLPs)?

How to solve hub location problems?

But ... isn't this related to ... ?

Where can I learn more?

What is a hub? Wikipedia

- Computing: Ethernet hub, computer networking device
- Transport
  - Transport hub, where traffic is exchanged across several modes of transport
  - Airline hub, an airport used as transfer point
  - Railway station
- Wheels: the central part of a bicycle wheel
- Organizations (Universities, social networks, more and more ...
- People
- Places
- Buildings (names of buildings)
- Fiction (comics, magazines, characters ...)
- Other uses: video games, TV channels, athletic teams,...

What is a hub? Dictionary

- A bucket
  (cube, physical container adapted to the human form, turbine blade of the turbine section of a gas turbine ...)

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- An activity center

What is a hub? for me

A center that facilitates service through a network

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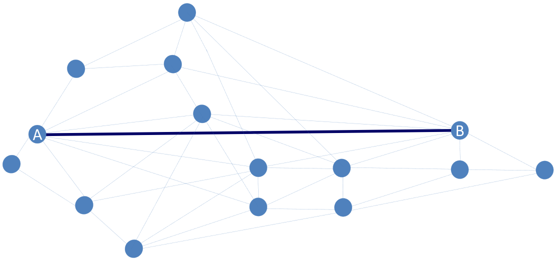
What is hub location about?

Efficient distribution of flows in networks

Point-to-Point distribution



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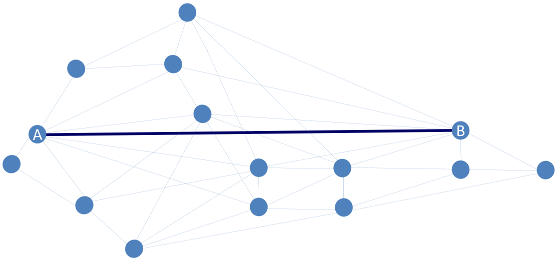

↑ Specific one-time orders

↑ Small structure requirements

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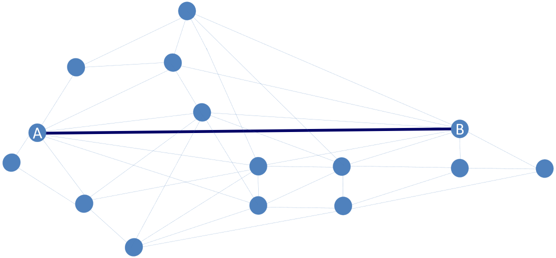

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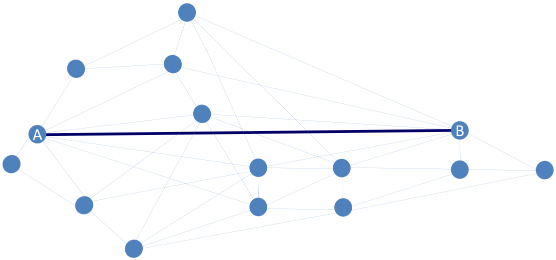

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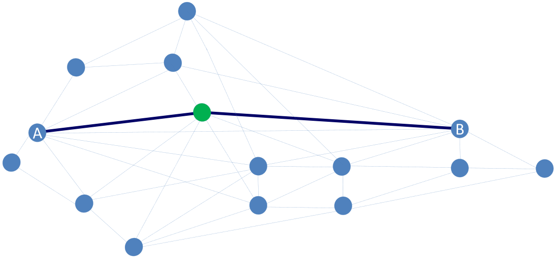
- Efficiency?

Hub-and-Spoke distribution



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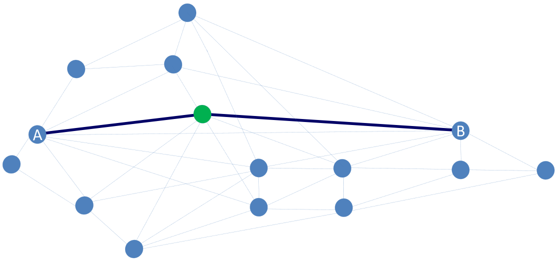

↑ Switching/sorting distribution centers for high throughput

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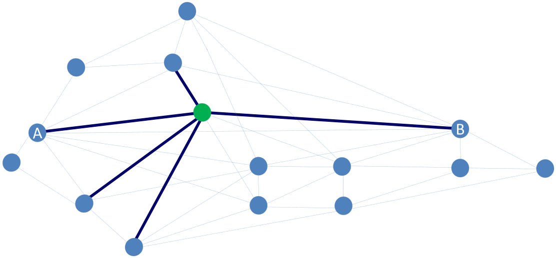

- ↑ Switching/sorting distribution centers for high throughput
- ↑ Consolidation (aggregation/disaggregation) of flows
- ↑ Privileged access

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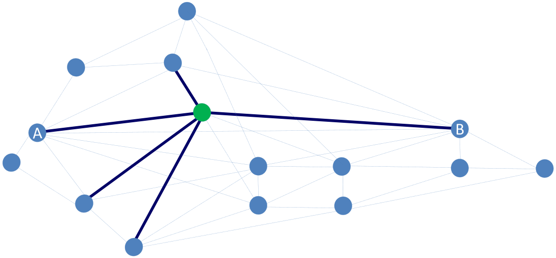

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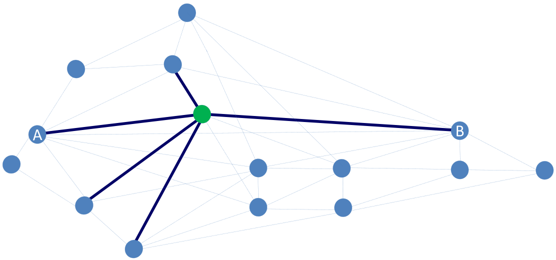

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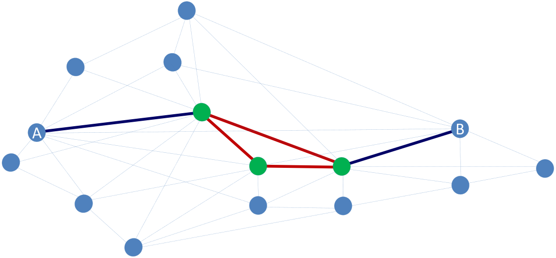
- Effective only if hub with capacity for large amounts of requests

Multi-hub distribution network



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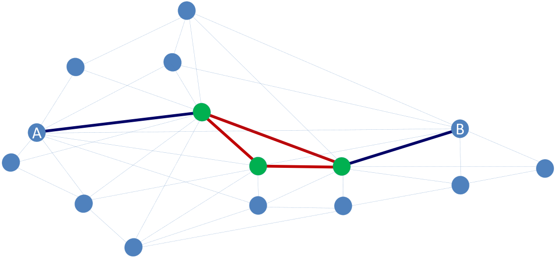

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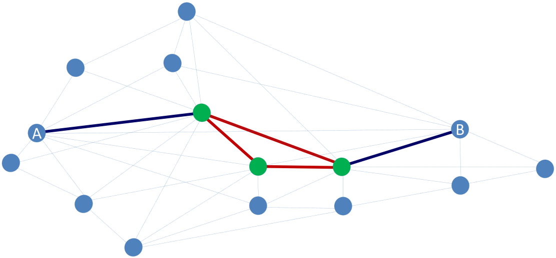

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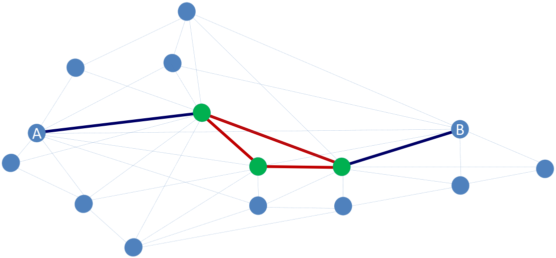

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- Smaller capacity requirements for hubs

When study hub location?

- There exists a large number of origin/destination (OD) pairs
- Flows can (must) be consolidated or re-routed at some facilities (hubs)

Why? The use of hubs helps ...

- ... reduce set-up costs
- ... centralize commodity sorting and handling operations
- ... achieve economies of scale on routing (distribution) costs

Decisions

- Location: Where to locate the hubs.
  Switching, sorting, aggregating, disaggregating

- Distribution: How to satisfy the demand $\Leftrightarrow$ How to send the flows
  Allocation/routing; Network Design

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Objective: Minimize routing costs

Minimize set-up costs costs

Some application areas

- Transportation, distribution, logistics ...

Facilities: Centers to handle flows of freight or passengers.

Routing: Movement of vehicles carrying the freight or passengers.

Links: Infrastructures (roads, railways, air, water).

Costs: Travel cost/time. Depend on distance traveled.

Objective: Transportation costs and travel times to serve a given demand.

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- Telecommunication, computer systems ....

Facilities: Hardware such as switchers, routers, concentrators to provide communication among ODs with demand.

Routing: Movement of electronic data.

Links: Physical links (cables, fiber links), or air (microwaves)

Costs: May or may not be significant routing costs.

Objective: Often focus on set-up costs (fixed costs for establishing the network)

When did it start? Over 25 years ago!

Campbell J, O'Kelly M (2012), Transportation Sci.

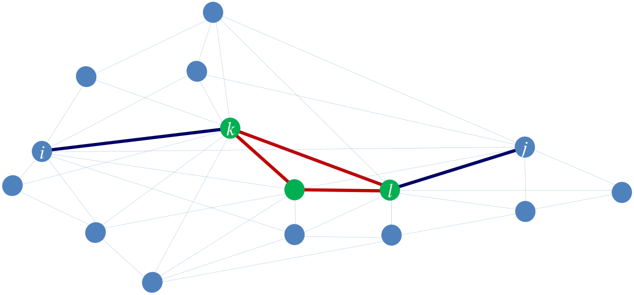
- O'Kelly ME (1986a) The location of interacting hub facilities, Transportation Sci. 20.
- O'Kelly ME (1986b) Activity levels at hub facilities in interacting networks, Geographical Anal. 18.

- Hakimi SL (1964) Optimal locations of switching centers and the absolute centers and medians of a graph. Oper. Res. 12.
- Hakimi SL (1965) Optimum distribution of switching centers in a communication network and some related graph theoretic problems. Oper. Res. 13.
- Goldman AJ (1969) Optimal location for centers in a network. Transportation Sci. 3.

Some notation

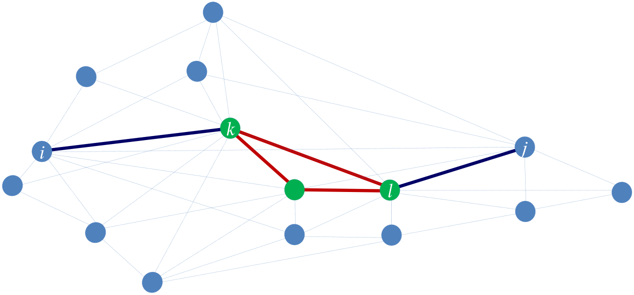
$G = (V, E)$:	undirected complete graph.
$c_{ij} \geq 0$:	distance (transportation cost) between i, j. (Symmetric + triangle inequality)
R:	set of commodities (O/D pairs).
W_{ij}:	flow (demand) that must be routed from i to j.
α:	discount factor for transportation costs between two hubs. $\alpha < 1$.
β:	collection weight factor (origin-hub transportation). $\beta > \alpha$.
δ:	collection weight factor (hub-destination transportation). $\delta > \alpha$.
$O_i = \sum_{j \in V} W_{ij}$:	Total flow that must be sent from i.
$D_i = \sum_{j \in V} W_{ij}$:	Total flow that must arrive i.

Modeling assumption: Origin-destination paths include at least one hub node



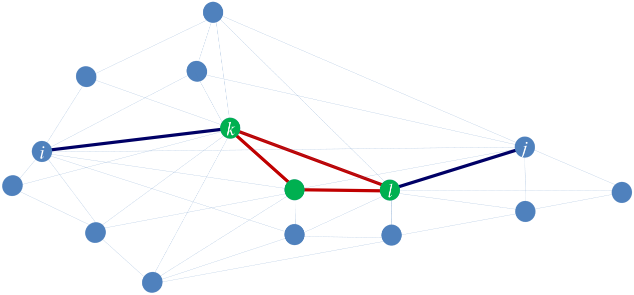
Modeling assumption: Origin-destination paths include at least one hub node

- Remove from demand set any flows large enough to be sent directly.



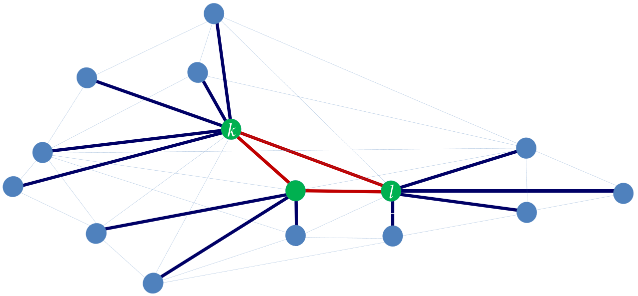
Modeling assumption: Origin-destination paths include at least one hub node

- If triangle inequality holds: all O/D paths include one or two hub nodes.



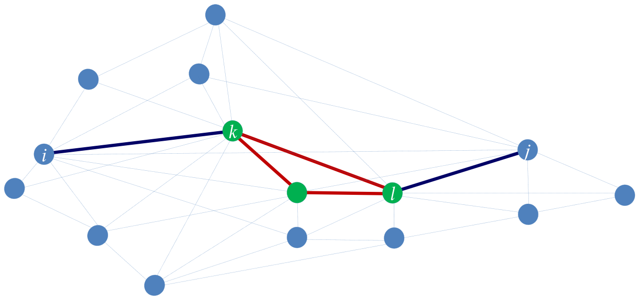
Modeling assumption: Star access networks

- Each non-hub node directly connected to a hub node.



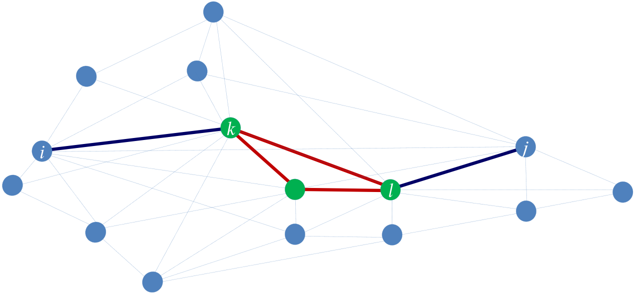
Modeling assumption: At least one hub node + △ + star access

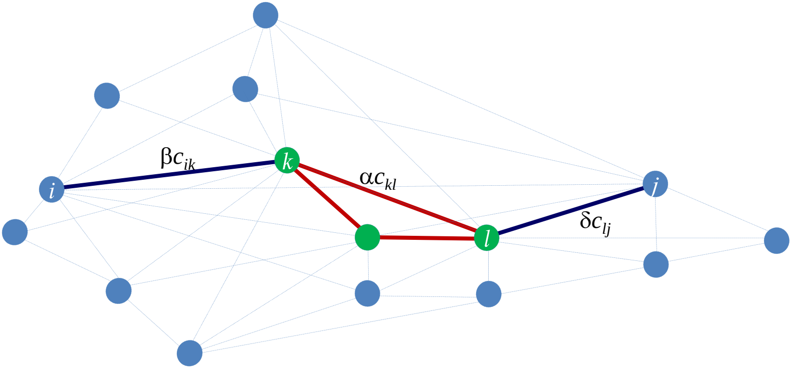
- All origin-destination paths consist of at most three arcs.



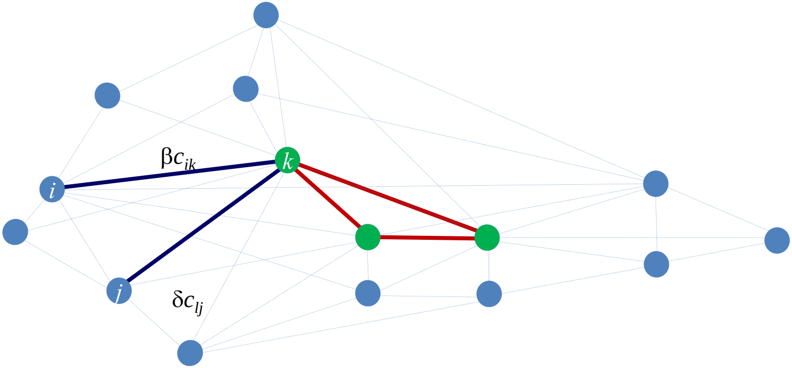
Modeling assumption: At least one hub node + △ + star access

- All origin-destination paths consist of at most three arcs.
  - *access arcs*
  - *hub arcs* (with reduced transportation cost)





$$d_{ijkl} = \beta c_{ik} + \alpha c_{kl} + \delta c_{lj}$$



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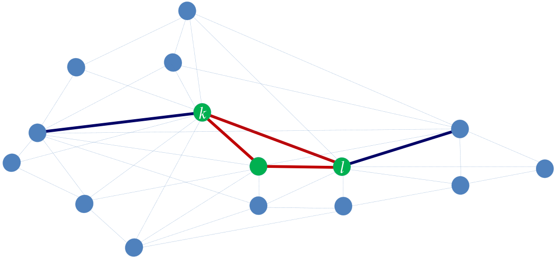
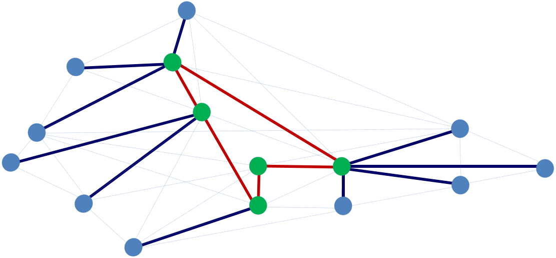
Complete vs incomplete hub networks



- Hubs induce complete graph
- For free if no hub set-up costs + triangle inequality
- Otherwise, it is a simplification
- In the general case it has to be imposed

Complete vs incomplete hub networks: Ring Star

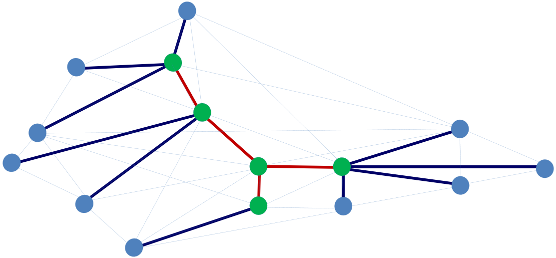
Labbé, et al. (2004), Labbé and Yaman (2008), Contreras and EF (2012)



- Hub nodes connected by a ring
- Each customer allocated to one hub

Complete vs incomplete hub networks: Tree of hubs

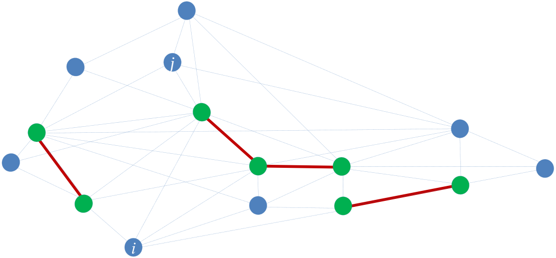
Contreras, EF and Marín (2009, 2010)



- Hubs define a tree
- Specific constraints

Complete vs incomplete hub networks: Hub arc location

Campbell, Ernst, Krishnamoorthy (2005a,b), Contreras and EF (2013)



- Endnodes of hub arcs must be hub nodes
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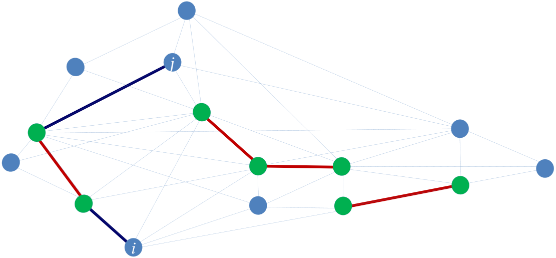

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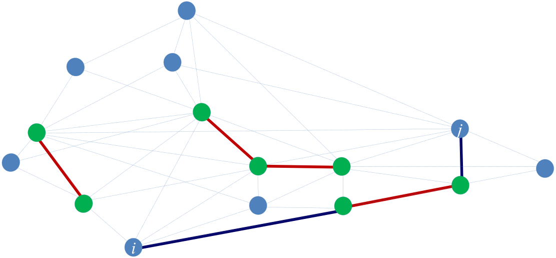

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Single vs. multiple allocation

- Single allocation: All the flow leaving a fixed vertex i goes through the same hub.
  Less-than-truckload trucking networks: each non-hub (end-of-line) assigned to one single hub (breakbulk)
  Telecom applications with star access: when reducing set-up costs.

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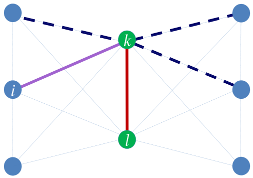
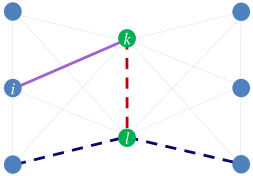
- Multiple allocation: Depending on the destination, flow may leave vertex *i* trough different hubs
  Passenger airline networks
  Telecom applications: when increasing reliability, or provide backups.

Single vs. multiple allocation





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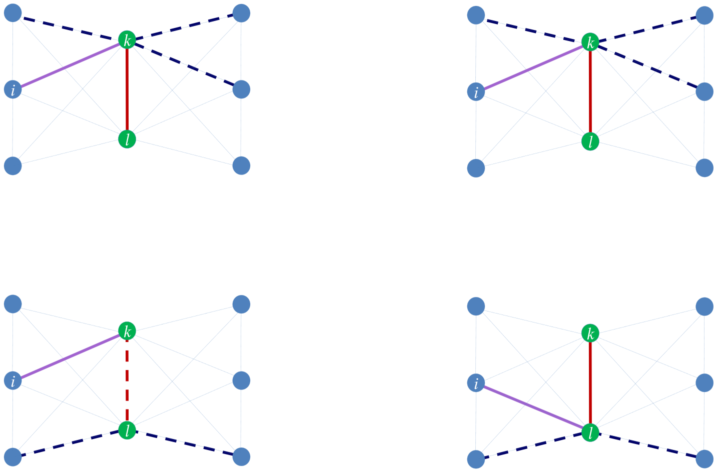

Two classical HLPs

- Open exactly p hubs
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  - Marín A, Cánovas L, Landete M, EJOR (2006).

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Decision variables: $z_{ik} = 1 \Leftrightarrow$ customer i assigned to hub k, for $i, k \in V$

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$$\sum_{k \in V} z_{ik} = 1 \qquad i \in V$$

$$z_{ik} \leq z_{kk} \qquad i, k \in V, i \neq k$$

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Multiple allocation p-hub median (Marín, Cánovas, Landete, 2006)

Decision variables. x_{ijkl} : fraction of flow from i to j on path $i-k-l-j$

$$\min \sum_{(i,j)\in R} W_{ij} \sum_{k,l\in V} d_{ijkl} x_{ijkl}$$

Allocation
$$\sum_{k,l\in V} x_{ijkl} = 1 \qquad i,j \in V$$

Relate x and z
$$x_{ijkk} + \sum_{l\neq k} (x_{ijkl} + x_{ijlk}) \leq z_{kk} \qquad i,j,k \in V$$

p hubs
$$\sum_{k\in V} z_{kk} = p$$

$$z_{kk} \in \{0,1\}, \qquad k \in V$$

$$0 \leq x_{ijkl} \leq 1 \qquad i,j,k,l \in V$$

Formulations with 4-index variables

$\uparrow$ Very tight LP bounds

$\downarrow$ (Too) many variables

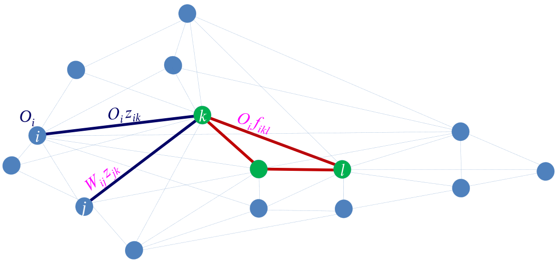
- Very high memory requirements
- Decomposition methods

Single allocation with 3-index variables (Ernst & Krishnamoorthy, 96)

f_{ikl}: Fraction of flow emanating from i routed via hubs k and l

Flow conservation for $i, k \in V$

$$\sum_{l \in V} O_i f_{ikl} + \sum_{j \in V} W_{ij} z_{jk} = O_i z_{ik} + \sum_{l \in V} O_i f_{ilk}$$



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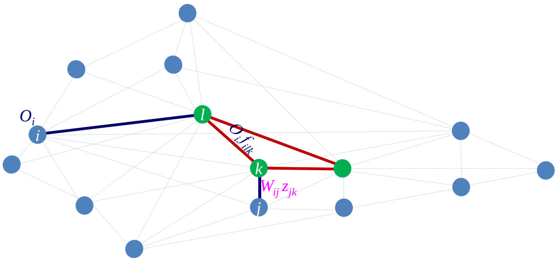


Relevant optimization criteria

- Routing (service) costs
  - Overall service cost (median problems)
  - Maximum service cost (center problems)
- Design costs
  - hubs set-up costs
    (easy to formulate with the z variables)
  - edges (connections) set-up costs
    (may require additional variables with multiple allocation)

Other issues

- Capacity
- Time issues
- Stochasticity
- Reliability
- Competitive
- Multicriteria
- Less restrictive modeling hypotheses / Alternative topologies
- ...

How to solve HLPs? HLPs are not easy to solve

- NP-hard.
- Tight formulations require many variables.
- Formulations with fewer variables can only solve small size instances.
- Medium size formulations very difficult to solve to optimality with general purpose solvers.
- What can one do?

How to solve HLPs? HLPs are not easy to solve

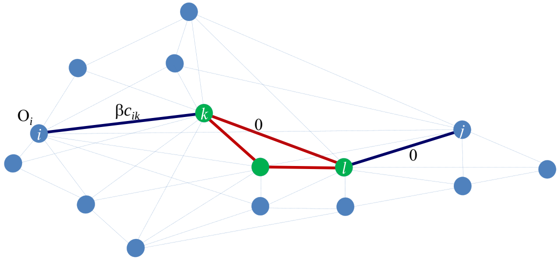
- NP-hard.
- Tight formulations require many variables.
- Formulations with fewer variables can only solve small size instances.
- Medium size formulations very difficult to solve to optimality with general purpose solvers.
- What can one do?
  - Develop tighter polyhedral formulations suitable for Branch&Cut
  - Develop alternative formulations
    (Based on radii - distance based, supermodular properties, ...)
  - Decomposition methods
    (Lagrangean relaxation, Column generation, Benders decomposition ...)
  - Heuristic methods
  - ...?

What instances can be solved?

Problem	Nodes	Technique	Who
Multiple p-hub	200	B&C (radii)	García, Landete, Marín EJOR (2012)
Single p-hub	400	VNS	Ilíc, Urošević, Brimberg, Mladenović EJOR (2010)
Multiple uncap.	500	Benders	Contreras, Cordeau, Laporte Oper. Res. (2011)
Single cap.	200	B&P	Contreras, Díaz, EF INFORMS JoC (2011)
Single hub-center	400	ACO + B&B	Meyer, Ernst, Krisnamoorthy C&OR (2009)
q-hub Arc	125	B&C (supermod.)	Contreras, EF Oper. Res (2013) (minor rev.)

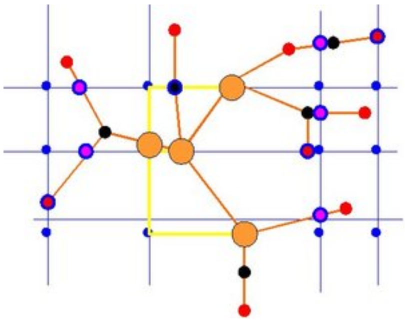
Hub Location & Facility Location

If $\alpha = 0$, $\beta = 0$, location is particular case (Contreras and EF, 2012)



Hub Location & Network Design

Connected facility Location (Gollowitzer and Ljubić, C&OR, 2011)



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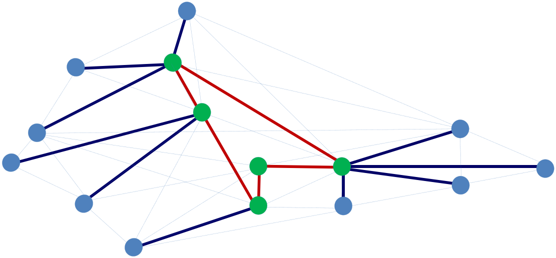
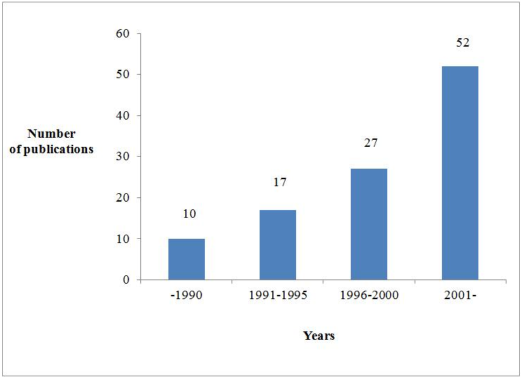
Hub Location & Network Design

Ring Star (Labbé, et al., Networks, 2004)

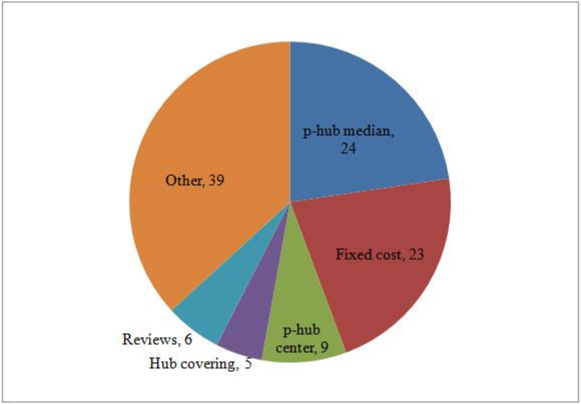


Number of publications



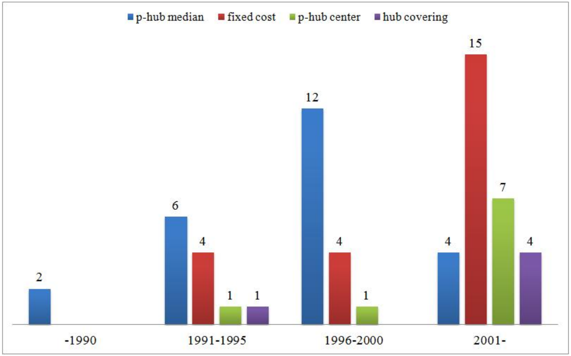
Source: Alumur & Kara (2008)

Topics of publications



Source: Alumur & Kara (2008)

Topics of publications



Source: Alumur & Kara (2008)

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- Campbell J, O'Kelly M (2012) Twenty-Five Years of Hub Location Research, Transportation Sci. 46.
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Muito obrigada!